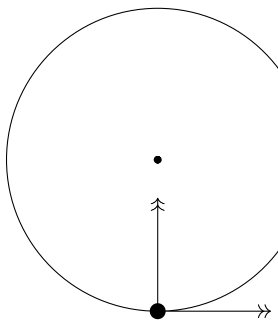
Horizontal circular motion with variable speed

Recall from earlier, that

Fact — An object moving in a circle has acceleration of magnitude $\frac{v^2}{r}$ or $r\omega^2$ directed towards the centre of the circle.

Suppose now we are moving with variable speed.



Fact — An object moving in a circle has acceleration of magnitude $\frac{v^2}{r}$ or $r\omega^2$ directed towards the centre of the circle and tangential acceleration of

Example

A car, of mass 1200 kg, is travelling on a roundabout of radius 50m. At the point P, it is travelling at 15ms^{-1} and its speed is increasing at 2ms^{-2}.

Find the magnitude of the resultant force on the car:

(a) at the point P.

(b) 5s after it has left P

Example

A car of mass 1000 kg is following a horizontal, circular path of radius 100m. The coefficient of friction between the tyres of the car and the road is 0.7. Determine the maximum possible rate at which the speed of the car can increase when it is travelling at 12ms^{-1}

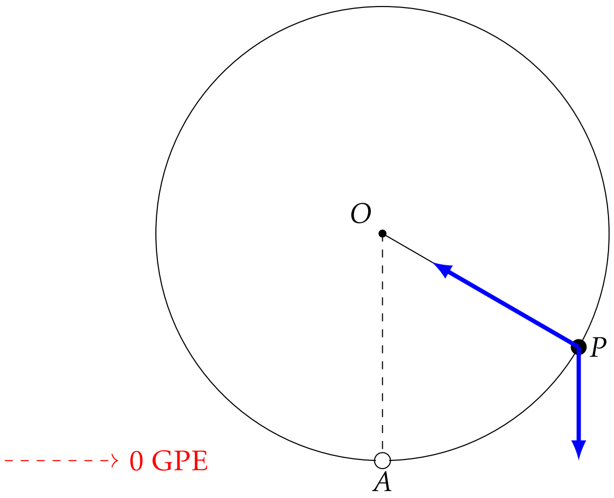
Motion in a vertical circle

Example

A particle P of mass 5 kg is suspended from a fixed point O by a **light inextensible string** of length 1m. The particle is projected from its lowest position at the point A, with a horizontal speed of 4ms^{-1}. When the angle $\angle AOP = 60°$ find:

(a) the speed of P

(b) the tension in the string



(a)

$$\text{COE}: \quad \frac{1}{2}\cdot 5\cdot 4^2 = \frac{1}{2}\cdot 5\cdot v^2 + 5g(1-\cos 60°)$$

$$\Rightarrow \quad v^2 = 4^2 - g$$

$$\Rightarrow \quad v = 2.49\,\text{ms}^{-1}\,(3\ s.f.)$$

(b)

$$N2(\nwarrow, radially): \quad T - 5g\cos 60° = \frac{mv^2}{r}$$

$$\Rightarrow \quad T = \tfrac{5}{2}g + 5(16-g)$$

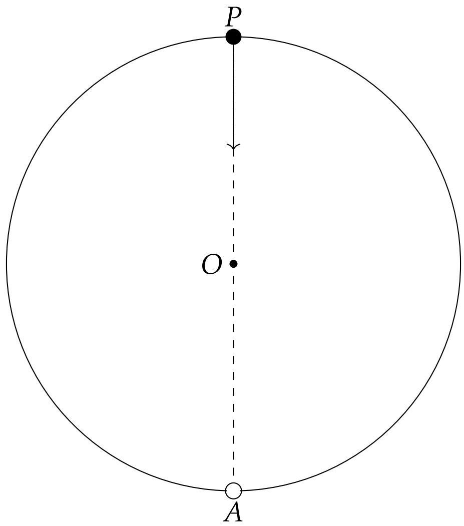
$$= 55.5\,\text{N}\,(3\ s.f.)$$

Example

A small bead of mass 3 kg is threaded onto a **smooth wire** in the shape of a circle, radius 2m and centre O. The circular wire is fixed in a vertical plane and the bead passes through the lowest point A on the wire with speed 10ms^{-1}. When the bead is at the top of the wire find:

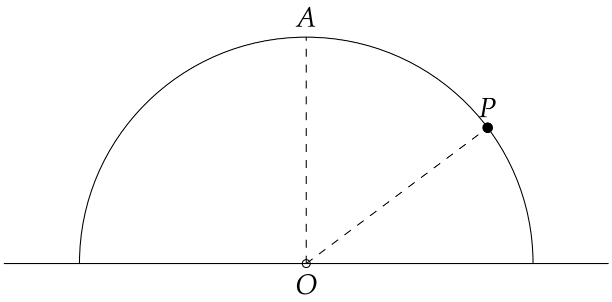
(a) the velocity of the bead

(b) the magnitude of the reaction between the bead and the wire



Example

A particle, of mass 5 kg, is placed at the point A at the top of a hemisphere, of radius 2 m and centre O. The hemisphere is fixed to a horizontal surface, as shown in the diagram. The particular is set into motion with an initial speed of $3\,\text{ms}^{-1}$. The particle leaves the surface of the hemisphere at the point B, find the angle $\angle AOB$



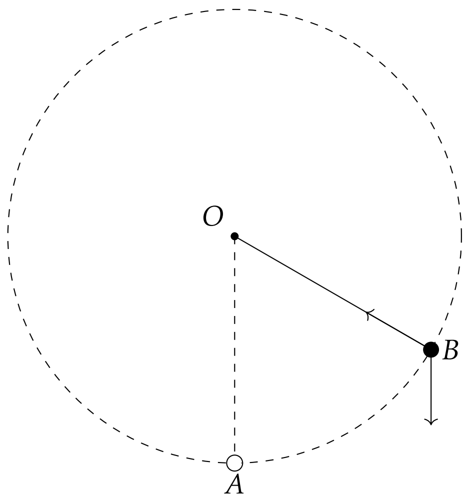
Conditions for a particle to perform complete circles

Example

A **light rod** of length 2 m is pivoted at one end, O, and has a particle of mass 8 kg attached to the other end. The rod is held vertically with the particle at A, directly below O, and the particle is given an initial horizontal speed $u\,\text{ms}^{-1}$. Find:

(a) an expression in terms of u and θ for the speed of the particle at point B where $\angle AOB = \theta$

(b) the restriction on u^2 if the particle is to perform complete circles

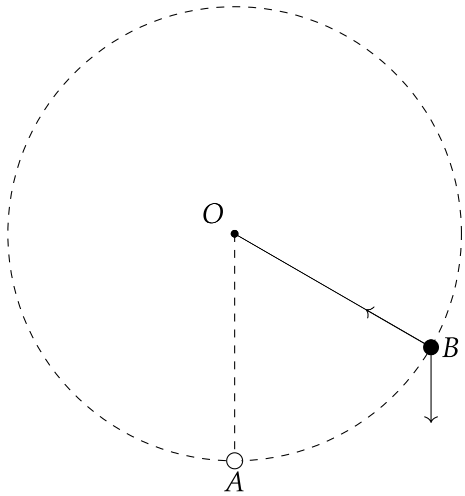


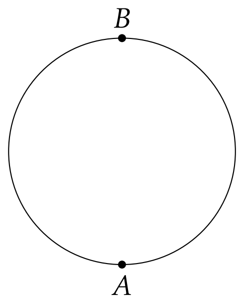
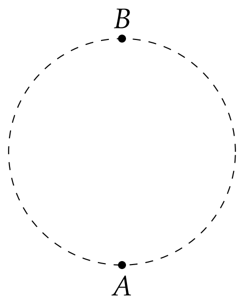
Example

A particle P of mass mass 8 kg is suspended from a fixed point O, by a **light inextensible string** of length 2 m, and the particle is projected with an initial horizontal speed $u\,\text{ms}^{-1}$. Find:

(a) an expression in terms of u and θ for the speed of the particle at point B where $\angle AOB = \theta$

(b) the restriction on u^2 if the particle is to perform complete circles



Setup	Conditions	
Bead threaded on a smooth wire		
Particle attached to light rod		
Particle attached to light string